Discovering Entropy

OBJECTIVES

- To explore different definitions of entropy.
- To simulate entropy using dice sets and analyze the outcomes using a histogram.
- To understand the history of entropy through the lens of historical physicists and chemists.

MATERIALS

Excerpts from *Max the Demon vs. Entropy of Doom*
A set of dice

READING & REFLECTION

1. There has long been discussion around what entropy is *conceptually*, not just mathematically. Here, we'll spend some time thinking about some different definitions you may encounter. This lab will be more of a problem-solving session than a traditional lab. Please write your reflections, observations, and data down in your lab book or on a separate sheet of paper--- these will be handed in and counted towards your grade for this lab.

2. Read the following thought experiment proposed by James Clerk Maxwell in 1867:

   > *... if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us. For we have seen that molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics.*

   With your group, consider the following questions:

   - Maxwell describes that individual molecules travel at different velocities, but their average is always uniform. What measurable quantity is he describing here?

   - Draw a set of diagrams that depict the experiment that Maxwell describes.

   - How does this thought experiment violate the second law of thermodynamics? What would you observe if the experiment could be carried out?

   This "finite being" that Maxwell describes here later was named "Maxwell's Demon" by Lord Kelvin, and the name stuck. This is still a problem that is discussed today as we research and untangle how we understand entropy in the context of the philosophy of science and theoretical physics.

3. On Canvas, read the excerpts from the graphic novel *Max the Demon vs. Entropy of Doom* by physicist Assa Auerbach and illustrator Richard Codor. We're jumping into the middle of the story, but you can grab the full copy from the library (it's on reserve for this course) or borrow my copy in the lab if you'd like to read the full work. Here's the synopsis:

    Max, a character based on the mythical Maxwell's Demon, is sent on a mission to help save earth from environmental disaster. First, Max must learn about energy, heat, and entropy from the historical giants of science. Along the way there is villainy and near catastrophe.

PART I: ENTROPY AS A MATHEMATICAL RELATIONSHIP

1. Entropy S is fundamentally defined by the equation
$$dS = \frac{dQ}{T}$$
where Q is an amount of heat in joules, and T is a temperature in Kelvin.

2. Using the differential definition of entropy above, consider the special cases of the first law of thermodynamics we considered in class. In each case, what would you expect the entropy to be? Assume an ideal gas and be as specific as possible.

    a. Adiabatic processes: $dQ = 0, dE_{int} = dW$

    b. Constant volume processes: $dW = 0, dE_{int} = dQ$

    c. Cyclical processes: $dE_{int} = 0, dQ = dW$

    d. Free expansions: $dQ = dW = dE_{int} = 0$

PART II: ENTROPY AS UNAVAILABLE ENERGY

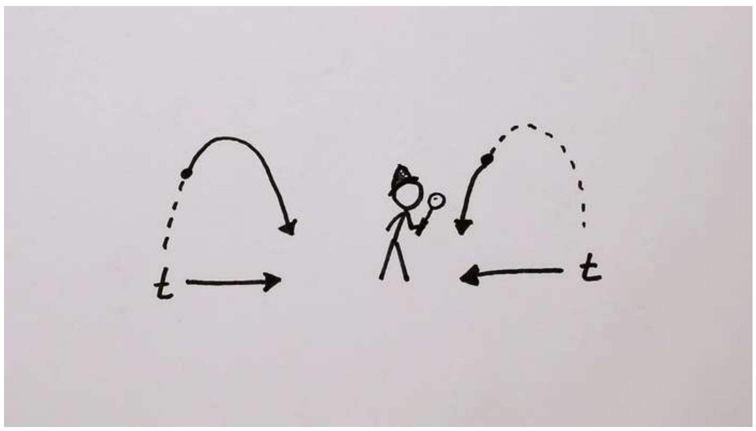
3. One of the earliest definitions of entropy came from Lazare Carnot and his son, Sadi Carnot (who is referenced in *Max the Demon*). In thinking about mechanical systems (for example, pulleys and inclined planes), Lazare Carnot observed that energy is always lost through friction or other dissipative forces. Sadi Carnot later extended this idea in his development of engines, and hypothesized that even in an idealized engine, some heat would always be lost.

    One way we can think about entropy then is as *unavailable energy.* Imagine a piston containing a gas at a certain temperature T_0. If thermal energy is added to the gas, raising its internal temperature to T, the gas will do work on the piston and start pushing it upwards. If you had the power to arrange the direction and velocity of the particles in the gas in order to maximize the efficiency of the gas pushing on the piston, how would you arrange them? How do you think they are arranged in reality? Sketch each situation as you discuss it with your group.

PART III: ENTROPY AS THE ARROW OF TIME

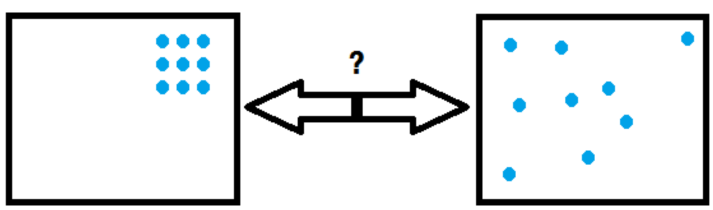
4. Entropy is also sometimes looked at as a way to anchor the direction that *time* flows. One of the yet unanswered questions in physics is, why does time have a direction? At the microscopic level, if time were to reverse, it is thought that most of the laws of physics would remain the same--- the mathematics that describe the microscopic level wouldn't change. However, at a macroscopic scale, we would immediately know if time were reversed.

   Many forces have a symmetry in time; for example, gravity is a time-reversible force. If you took a video of a ball being thrown up in the air and being caught again, the physics of that experience would not look very strange if you watched it in reverse. But, if we were to drop a ball on the ground and record a video of it bouncing, the physics of that experience would look odd if we were to watch it in reverse. What other examples of time-symmetric forces can you come up with? What examples of time-asymmetric forces (or phenomenon) can you come up with?



MinutePhysics (https://www.britannica.com/science/arrow-of-time/images-videos#/media/1/1324069/204018)

5. Given the two snapshots of a system of a gas in a closed and insulated box below, which way is time flowing? Justify your decision with your group. How does this connect with the definition of entropy as chaos or as unavailable energy?



Which direction is time flowing?

https://energyeducation.ca/encyclopedia/Entropy

PART IV: ENTROPY FROM A STATISTICAL PERSPECTIVE

6. Entropy can be described as the amount of disorder or chaos in a system, but this doesn't quite get at some of the nuances of what entropy represents. In this activity, we'll explore the statistical side of entropy. In **statistical mechanics**, the thermodynamics we are investigating in this course are explained through the motion of atoms and molecules (when thermodynamics was first being developed, the concept of an atom had yet to be developed).

Lab

Consider the coin toss described in the excerpt from *Max the Demon*. A single coin toss carries a random probability, but if we flip the same coin many times, we reach a predictable result--- half the time the coin will land on one side, and half the time it will land on the other. What other examples can you come up with where an individual outcome is unpredictable, but a large number of outcomes *is* predictable?

7. In statistical mechanics, entropy S is defined in terms of the number of **microstates** W of a system,

$$S = k_B \ln W$$

8. According to what you read in *Max the Demon,* what is a microstate?

9. Consider a single dice (1d4, 1d6, or 1d8). What is the probability of any one number being rolled?

10. Take a pair of four-sided, six-sided, or eight-sided dice (2d4, 2d6, or 2d8). Using your method of choice (whiteboard, paper, Excel, etc.) tabulate all the possible values you can get by adding the digits of the two dice together. Determine the *multiplicity (*or *degeneracy)* of each value: how many different ways can you roll each possible value? What do you think the most likely values are? Which values are the least likely?

11. Using the multiplicities, calculate the probability for each outcome you determined in the previous step. If the dice behave exactly as you expect them to, what would a histogram of 64 trials look like? 128 trials? By hand or in Excel, create a histogram showing the theoretical prediction of the dice behavior.

12. Using the two dice you chose, roll the pair of dice 64 times. Record the sum of each roll.

13. Plot a histogram of your results (either by hand, or in Excel). What is your most common value? Is it what you expected? Check for any unexpected behavior in your histogram. Would you expect the results to change if you took more data?

14. Roll your pair of dice another 64 times (perhaps let your partner do it this time). Record the sum of each roll.

15. Plot your combined results from steps 7 and 9 in one histogram. What changed?

16. Ludwig Boltzmann was the first to define entropy in terms of microstates (his equation is engraved on his tombstone in Vienna), and through this determined that a highly ordered state (for example, one with all molecules travelling in the same direction) was the most improbable configuration of a system. In other words, the more "disordered" or chaotic a system was, the more likely it was to occur.

[Daderot](#) at [English Wikipedia](#)

How does this connect with the other definitions we've discussed? How much energy would be unavailable to be used as work in a highly ordered system?

CONCLUSION

To finish the exercise, fill out this short survey on how you found the incorporation of a graphic novel into this lab activity: https://forms.gle/QoBgzjZwFGtEr99Y8